U N C E R T A I N T Y

A Statistical Perspective

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Can the topic of statistics be approached by photography or can photography be approached by statistical methods? What does photography has in common with statistics? Historically, both developed around the same time in the nineteenth century and in both cases chance plays a crucial role.

The reason why we need statistics to find answers to many real world questions is because these questions cannot be answered by *yes* or *no*. Let us have a look at the topic of the distinctive aesthetics of the *golden section*. The golden section is a certain ratio of two quantities a and b. If a is approximately 1.6 times the size of b, a and b are in the golden section, because:

$$\frac{a}{b} = \frac{a+b}{a}$$

The exact ratio is defined as:

$$\frac{1+\sqrt{5}}{2} = 1.61803\ldots$$

The number often appears in nature and geometry and has surprising mathematical properties. It has fascinated not only mathematicians but biologists, artists, architects, psychologists, and many others since thousands of years. Psychologists postulated that the golden section is related to the perception of beauty. In visual arts the golden section is often observed and taught as a basic principle of composing a picture: principal elements of a composition are rarely positioned in the centre but more often in the upper, lower, left or right third.

But is the golden section really perceived as most aesthetic? In order to answer this question a statistician would design an *empirical study* where people are asked either to position an object within a frame (*to produce*) or *to choose* between pictures. First, we have to decide whom to ask: students, experts, elderly, Caucasians, the general population? The decision depends on the conclusions we want to draw.

After carrying out the study and collecting the data, the statistical analysis aims to find out if the respondents prefer a certain positioning or if there is no distinction. The research question is only interesting, if the result is not obvious. In the very unlikely event that everybody prefers one option or positions the object exactly at the same spot, we do not and even cannot apply a statistical analysis. There is no uncertainty left to explain. Challenging (and the most common) problems are those where preferences are not obvious but at the same time they are not completely evenly distributed. There seems to be "some" trend.

Statistical methods try to distinguish between "some" trend that happened by chance and a *significant* trend that has a high probability to be real and to be reproducible in other studies.

Hence, the statistical analysis tells us what to conclude:

1. the golden section is preferred

   or

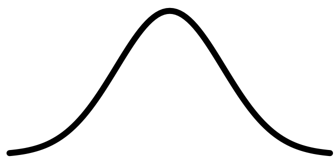
2. there is no evidence for a preferred position

The conclusions are based on a careful interpretation of the results considering validity, generalizability, bias, human errors and influences, measurement problems, desirability, wishful thinking and lack of causation.

Variation. The underlying principle that masks our clear yes-no signal with noise is called variation. The *Yes* and the *No* both become blurred. Imagine an overlay of hundred images of people touching a door handle before they press it: everybody's hand looks slightly different and everybody touches the handle in a somewhat distinctive way. We will not end up with a picture evenly covered with hands and it will not be a sharp picture showing a single hand. Most likely there will be a central position where most people press the handle, some might touch it more to the left, others more to the right. There might be one or two people with a very unusual "touch". The same is true for the difference in hand shape and size. There will be an average hand size for men and women, respectively. Some people will have extraordinary large hands, others will be rather small.

In statistics, the distribution of a feature, like hand size or door handle position is described by a *frequency distributions*.

A very common distribution is the *Gaussian normal distribution* which is bell-shaped:



The bell shape implies that observations that are average are more common than extremely small or large observations. The door handle example demonstrated that this is a common behaviour. The reason lies in the fact that usually many factors influence an outcome: some positively, others negatively. Very rarely, all factors have only a positive or only a negative impact. Most of the time it is a mix that leads to an average outcome.

The size of our hands is influenced by many genetic factors, by our nutrition but also by our behaviour: if we have used our hands for hard manual work, for a certain sport, for playing a music instrument. Each of these factors is again a product of many different influences. As long as there is not a single outstanding large factor that overrules or dominates all others, for example if we have a rare gene mutation or a severe accident, hand size will be normally distributed.

In real world situations, we typically observe a mix of variation and regularity, of noise and signal. When we look close enough, even objects that seem completely regular and geometric exhibit some degree of deviation. Others vary a lot, but we can still observe a regular pattern. It is not an arbitrary process.

Natural phenomena – a growing plant, a walking child, a column of steam, a mountain ridge, a tall tree, a swarm of birds, a rough sea – they follow rules and laws but to a certain degree, they are individual. The exact shapes are unpredictable and cannot be reproduced. But this individuality or variation or randomness is not arbitrary. The deviation of a tall stem from a straight line has a certain rhythm, balance and harmony. Two plants do not grow identical because of a great number of environmental factors: wind, water, light always act differently. *Wabi-Sabi*, the Japanese aesthetics and world view of imperfection, explicitly appreciates deviations from clear and geometric forms.

If we would know all factors that influence a phenomenon, we might be able to determine its exact shape. If this is possible in theory, if our world is deterministic or probabilistic, engaged philosophers for thousands of years. In practice, we rarely face phenomena that we can fully explain, at least not with reasonable effort. There are always some unknowns, chance or randomness or unexplained variation.

Chance – *a force that causes things to happen without any known cause or reason for doing so* – is a quintessential concept of statistics and plays a role in photography as well, a larger role than in other forms of art. This was recognized from the beginning of photography until very recently. Uncertainty is immanent when analysing, documenting, discussing our world. Photography as well as statistics are concerned with empirical, i.e. based on what is experienced or seen rather than on theory, problems and want to provide insights and a deeper understanding of these phenomena.

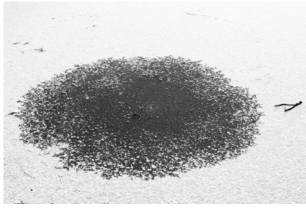
Chance in photography is an ambivalent issue. It opens up the possibility for a novice to end up with a handful of good photographs, which sums up to a large amount of good photographs when considering the number of amateurs. In the case of masters of photography, chance is blurring their virtuosity. Chance breaks the chain between cause and effect. In statistics, in so called randomized experiments, this is used to unlink the effect of an intervention from confounding factors.

If photography would only be a product of chance, the chance of producing a book full of good picture would decrease dramatically. Photographers reduce the role of chance in many ways: by their skills, their intuition and experience but also by editing, sequencing and arranging the picture. Some photographers stage their compositions extensively. Digital image processing is another way to reduce chance. In this aspect, image processing is much closer related to painting than photography.

But the element of uncertainty that is introduced into photography is often appreciated. Chance can be interpreted as arbitrariness but it can also produce a point of culmination where all stars suddenly align, the *decisive moment* in photography that Henri Cartier Bresson famously expressed as: "To me, photography is the simultaneous recognition, in a fraction of a second, of the significance of an event as well as of a precise organization of forms which give that event its proper expression."

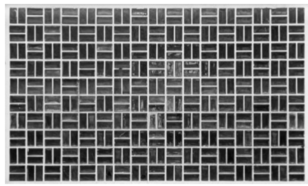
Sensitivity

property of a test: correctly identifying individuals that have the condition which is tested for.



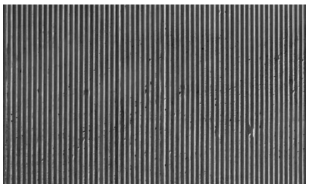
Specificity

property of a test: not identifying individuals that do not have the condition which is tested for.



Bivariate

statistics involving two variables, not necessarily independent of one another.



Univariate

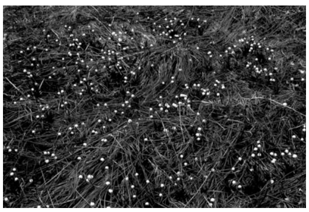
statistics involving one variable.

Homoscedasticity

constant variation for groups of observations, e.g. trees.

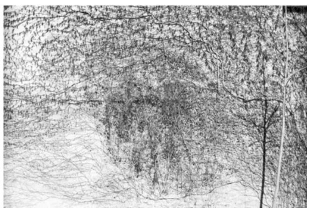
Heteroscedasticity

different variation for groups of observations, e.g. paint on pants.



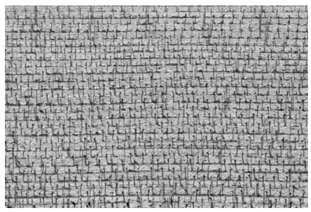
Independence

the occurrence of one event does not affect the probability of the other.



Dependence

the occurrence of one event does – to a certain degree – affect the probability of the other.



Variance

measure of how far a set of observations spread out from their mean.

Cross-Sectional

data collection at a single point in time.

Linearity

relationship between measures that can be described by a straight line.

Random Walk

a succession of random steps.

Optimization

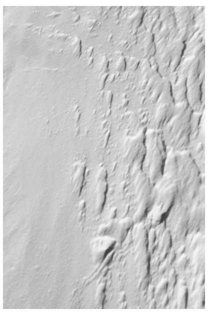
selection of a "best" element with regard to some criteria from a set of alternatives.

Convergence

sequence of unpredictable events that settles into a pattern.

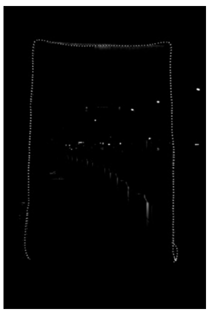
Parsimony

the principle of using the least explanations to solve a problem.



Discretization

categorization, i.e. converting a continuous measurement into a discrete one.



Deviance

measure of how good data fit to a model, e.g. a line.

Outlier

an observation that is distant from other observations.

Non-Normality

data that does not follow a normal distribution.

Kurtosis

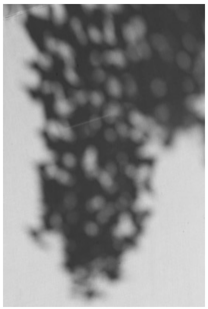
measure of the shape of a distribution, describing its steepness or tailedness.

Multiplicity

simultaneously testing of several hypothesis.

Sphericity

a measure of how spherical an object is.

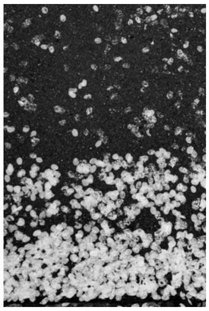


Unimodality

existence of a single highest value.

Bimodality

distribution with two peaks.

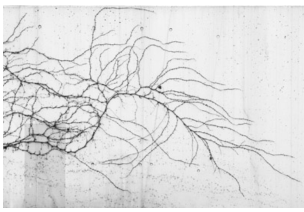


Skewness

measure of asymmetry of a distribution.

Correlation

the occurrence of one event affects to a certain degree the probability of another event.



Exponentiality

degree of being exponential, i.e. the growth rate being proportional to the current status.

Causality

connection between two characteristics, where one – the effect – is partly dependent on the other – the cause.